SFT Energy 2

CALIBRATION

Introduction

The determination of the energy of an electron or photon using the SFT information is described in the memo "SFT Energy 1". Using the emulsion modules as passive converters, the SFT hits and pulse height are used as the energy sampling detector as in standard calorimetry. The "SFT Energy 1" Monte Carlo simulations were used to form a first calibration in response to electron showers generated at various energies, with both transverse and longitudinal development parametrized for the system. In this memo, we use neutrino interaction data, from events selected to have well-defined and relatively isolated showers in the SFT and calorimeter.

Procedure

1) Events are selected using the requirements:
2) A well-defined SFT shower, isolated in at least u or v
3) An interaction vertex in Module 2 or 3
4) A separated calorimeter cluster with the shower axis or electron SFT track
5) At least 5 GeV of cluster energy

From Periods 3 and 4, ten events were selected with a vertex in Module 2 and five events were selected from Module 3. Module 4 vertices are rejected from this analysis as there is no shower development in the SFT, and almost all the energy is recorded in the calorimeter. Module 1 vertices are rejected for the opposite reason: very little energy remains in the calorimeter.

The calibration check of the energy using the SFT information alone, E_{SFT}, with the calorimeter energy, E_{cal}, consists of correcting E_{cal}

1

for losses in the emulsion targets and comparing the two, independent, results. In practice, one would add E_{SFT} to E_{cal} to get the best results. The "calibration" procedure used in "SFT Energy 1" used only 1 to 20 GeV electrons, beginning in Module 1. Therefore little energy remains in the calorimeter, and adding E_{cal} is a small correction. In the sample taken from data, where electron identification must be clear, the energies can be much larger, so E_{SFT} will underestimate the energy except for the lowest energy data.

Algorithms for Energy Calculation

The procedures adopted, in part, from the "SFT Energy 1" memo are reiterated here and expanded to include information from shower development. The outline for the calcuation is :

1) Get the calorimeter cluster positions for E_{clus}> 2 GeV
2) Form the vector connecting cluster position and vertex position
3) Get an estimate of the initial electron energy from cluster energy
4) Determine the width of the "road" in the SFTs for summing hits
5) Sum all the hits in the SFT(u,v) in along this road by stations
6) Calculate the energy estimate using SFT data
7) Calculate the total estimated initial energy adding cluster energy

Step 3) is given as :

$$E_{cal} = \frac{E_{clus} + 2.50}{0.97 - 0.0929x} \text{ GeV}$$

The 90% CL in this estimate is given as :

$$E_{90} = 0.055 + 0.066\,x$$

where x is the fractional radiation length from the vertex to the calorimeter, and depends on data Period. It assumes a radiation length of 3.3 cm for all E/B targets, 2.4 cm for ECC200 targets, and 3.5 cm for the bulk target. The value of x also includes the $1X_0$ lead sheets downstream of the targets.

Step 4) is given as :

$$width = \frac{0.1}{\log(E_{cal})} \text{ cm}$$

Step 6) is given as :

$$E_1 = 5.6p_1 \quad \text{GeV}$$

$$E_2 = \begin{cases} 0.55p_2 - 0.46 & p_1 < 10 \\ 1.44p_2 - 9.1 & p_1 \geq 10 \end{cases}$$

$$E_3 = 0.54p_3$$

$$E_4 = \begin{cases} 1.1p_4 - 0.31 & p_4 < 5 \\ 0.53p_4 + 2.6 & p_4 \geq 5 \end{cases}$$

where p_n is the summed pulse height for relative station n, within the road determined from the previous steps. The relative station is defined as the station minus the station where the vertex is located. The p_n are

```
if(E₄ = 0)
    if(E₃ > 0)
        E_SFT = (E₂+E₃)/2
    else
        E_SFT = E₂
    end if
else
    if(E₂ < (E₃+E₄)*0.3 )
        if(E₃ < E₄*0.6)
            E_SFT = E₄
        else
            E_SFT = (E₃+E₄)/2
        end if
    else
        E_SFT = (E₂+E₃+E₄)/3.
    end if
end if
```

Box 1. The algorithm for computing SFT energy from data in the four possible stations.

normalized to a mip in the SFT or 1620 counts. To incorporate the

shower development, the following procedure in Box 1 was used. To see how the flow of the procedure works, assume an interaction with an electron track is located in Station 1 (most upstream). The information from the first station is ingored, as there is little shower development. If there is no information in the last station, 4, then just use the average of energy values derived from stations 2 and 3. Note that in the case of full shower development (Station 4 has shower information), the energy assignment depends on where the maximum of the shower is located.

Results

Part A

From the results of data, whose distributions are shown in Appendix A, the SFT response, as modelled in the Monte Carlo, is described in terms of the parameters, p_n,to reproduce the initial energy of the electron. In the MC this reproduction of energy has a width of 17%. The parameters are determined specifically for energies < 15 GeV, since for higher energies, there is significant leakage of energy from the SFT system alone (keep in mind the events are generated at $z=0$). At higher initial energies, the calorimeter energy needs to be added to the SFT estimate. So, in this way, the SFT response is "calibrated" in energy, and represents a lower limit in the resolution to the response of the SFTs. Of course the MC includes effects such as shower development fluctuations and pulse height distributions (as determined from Reinhard's anaysis), but it cannot describe all possible sources of noise including random hits and superposed tracks.

Part B

The best calibration procedure would use an understood beam of electrons in the spectrometer, but this is not possible. It is possible to collect a set of events, neutrino interactions, which include identified electrons. We assume for the results, that the contamination of the electrons is negligible and that the calorimeter response is uniform and energy-calibrated.

The cluster energy for the electron is assumed to be properly associated, but will need to be corrected for energy that is absorbed or lost

before reaching the calorimeter. This process is well understood in the MC data. However it does introduce another smearing effect, which grows worse at greater depth (in radiation lengths) and lower initial energy. The width of the calorimeter response for 20 GeV electrons is 25% at $3X_0$, but is more than 45% at $6X_0$, and at larger depths, past shower maximum, the response is not symmetric and there is a significant probability that no calorimeter energy is recorded. Therefore, the measured resolution from Module 2 data will be much worse than Module 3 data, since the calorimeter corrected response is so broad. Nevertheless, this data can provide a calibration in the sense of the mean of the response. The measured width or resolution will be an overestimate of the true value.

The data in Table 1 demonstrate that the width of the response is much worse for Module 2 data compared to Module 3 data (45% vs. 18% which is near the lower limit predicted from the MC "calibration"). Another feature displayed in the data is that, on average, more energy is recorded in the calorimeter than in the SFTs for Module 3 data, and the reverse for Module 2 data. The data in Module 3 have depths ranging $3.1 < X/X_0 < 5.6$ and for Module 2 the range is $5.6 < X/X_0 < 7.1$. Except for the highest energy electrons, the depth for Module 2 data is beyond shower maximum, and relatively poor.

Event 3024-30175

The electron track is unambiguous in its identity, and is well isolated since there are only 3 charged tracks in this event. The number of hits agree in number in both views. There is no corresponding energy in the calorimeter (<0.5 GeV), which is not unusual for a shower of this energy and depth. The expected energy resolution is $\sigma_E/E = 0.20$ (or 0.33 for 90% CL). As the decay angle is 0.093, the estimated p_T using the SFT/Cal energy is 830±290 MeV/c.

p_1	p_2	p_3	p_4	z_{vtx}(cm)	X/X_0	E_{SFT}	E_{clus}	E_{cal}	E_{TOT}
2	3	14	14	+0.034	6.79	8.9±3.2	<0.5	<8.6	8.9±3.2

Table 2. Parameters and estimates of energy for the electron in event 3024-30175.

Event 3333-17665

The electron is not identified in the emulsion as such, but the track is aligned with a well-defined shower in the SFT as well as pointing to a cluster in the calorimeter. This cluster alone has an energy of 4.8 GeV, and there are two other blocks probably associated with this electron (another 1 GeV). The decay angle is 0.013, so using the SFT/Cal energy estimate, p_T = 390±70 MeV/c.

p_1	p_2	p_3	p_4	z_{vtx}(cm)	X/X_0	E_{SFT}	E_{clus}	E_{cal}	E_{TOT}
0	27	31	53	0.274	7.2	25.5±5.1	4.8	24.0	30.3±5.2

Table 3. Parameters and estimates of energy for the electron in event 3333-17665.

MC SFT Energy

3 GeV

ID	91
Entries	46
Mean	2.221
RMS	1.730

Sta 1

ID	91
Entries	50
Mean	5.628
RMS	2.598

Sta 2

Nhits

ID	91
Entries	50
Mean	5.304
RMS	2.223

Sta 3

ID	91
Entries	50
Mean	2.833
RMS	1.505

Sta 4

Nhits

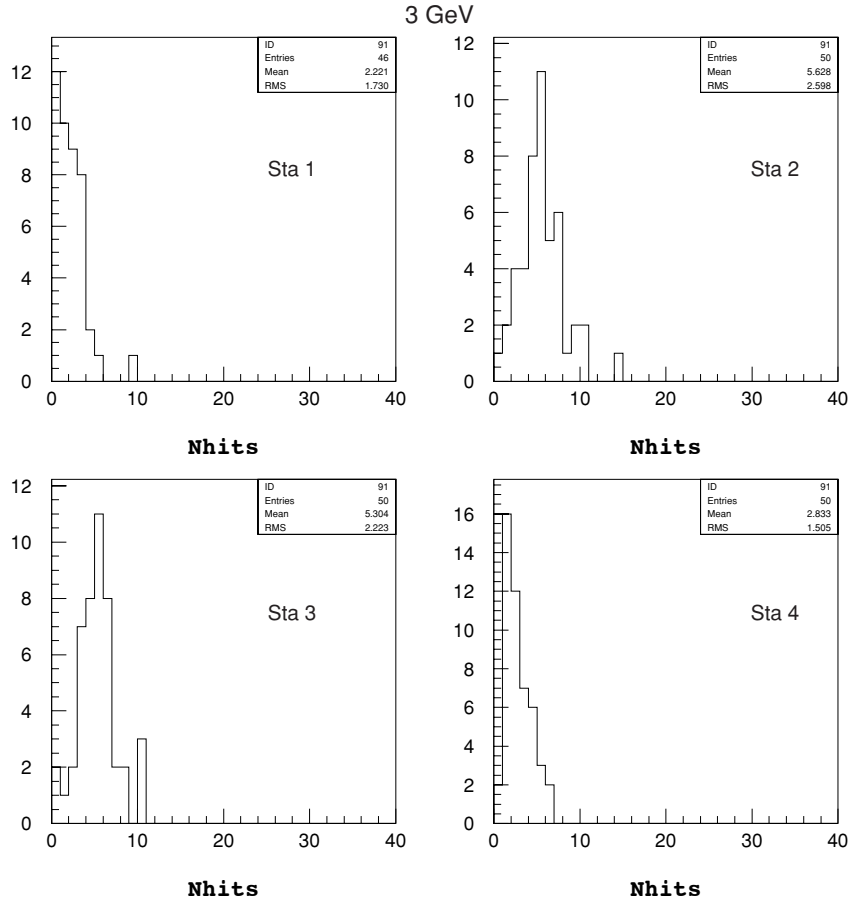**Figure 1** The number of SFT hits for Monte Carlo 3 GeV electrons originating at z=0 for the four detector stations
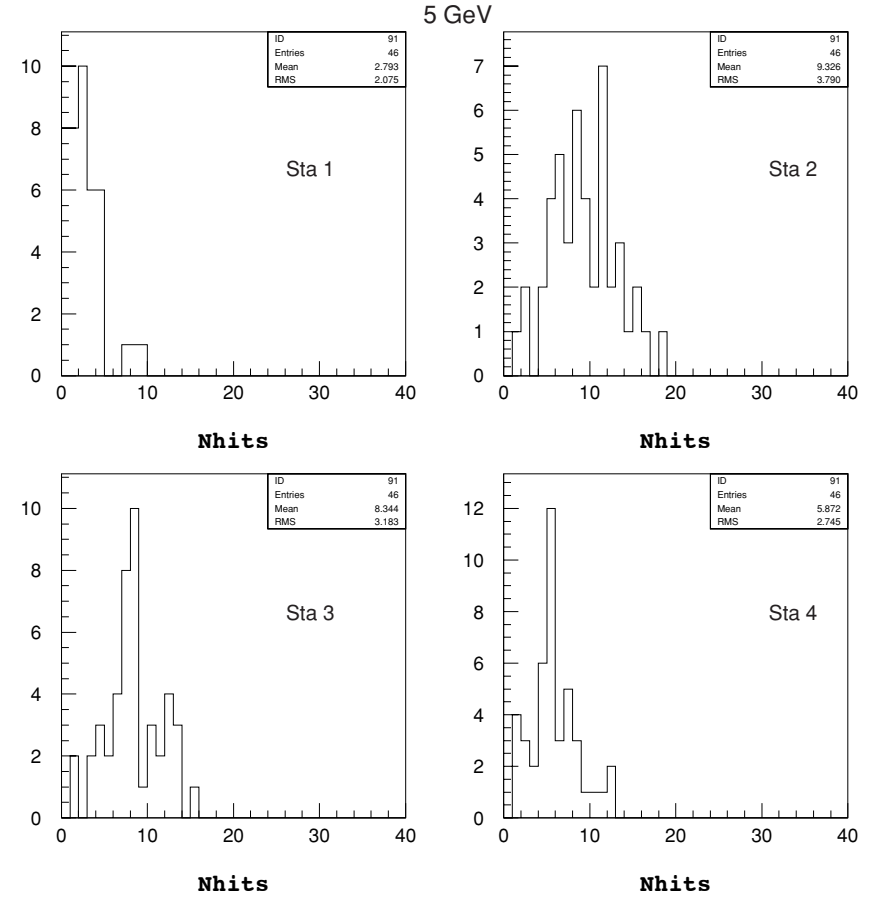
MC SFT Energy

5 GeV

ID	91
Entries	46
Mean	2.793
RMS	2.075

Sta 1

ID	91
Entries	46
Mean	9.326
RMS	3.790

Sta 2

Nhits

ID	91
Entries	46
Mean	8.344
RMS	3.183

Sta 3

ID	91
Entries	46
Mean	5.872
RMS	2.745

Sta 4

Nhits

Figure 2 The number of SFT hits for Monte Carlo 5 GeV electrons originating at z=0 for the four detector stations
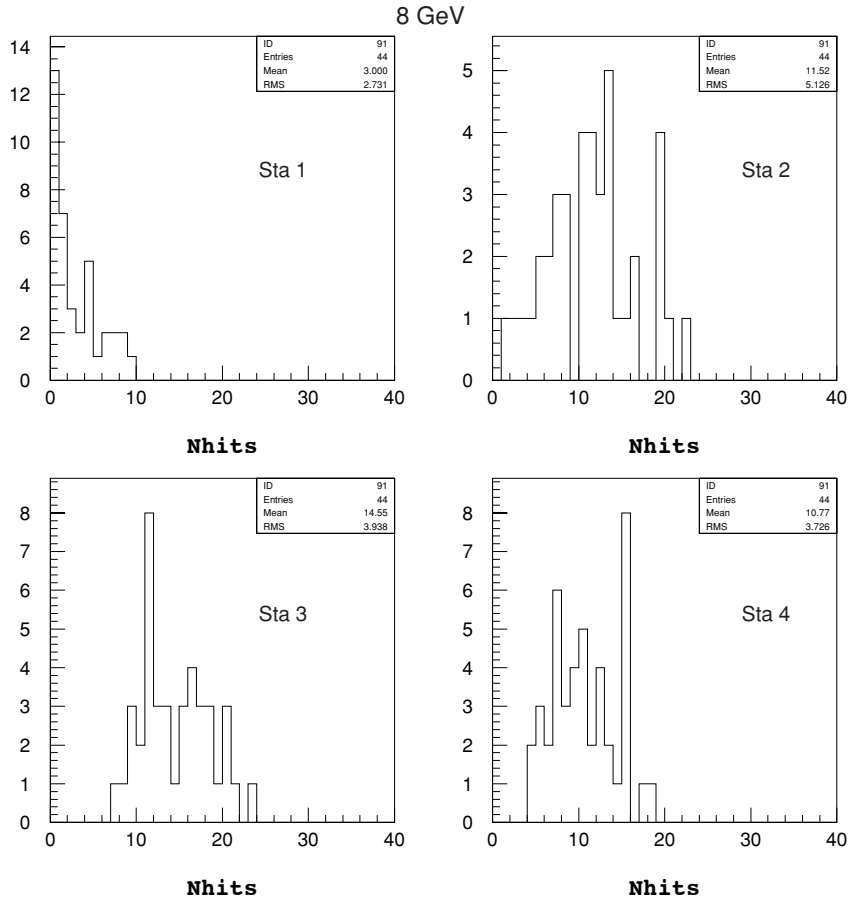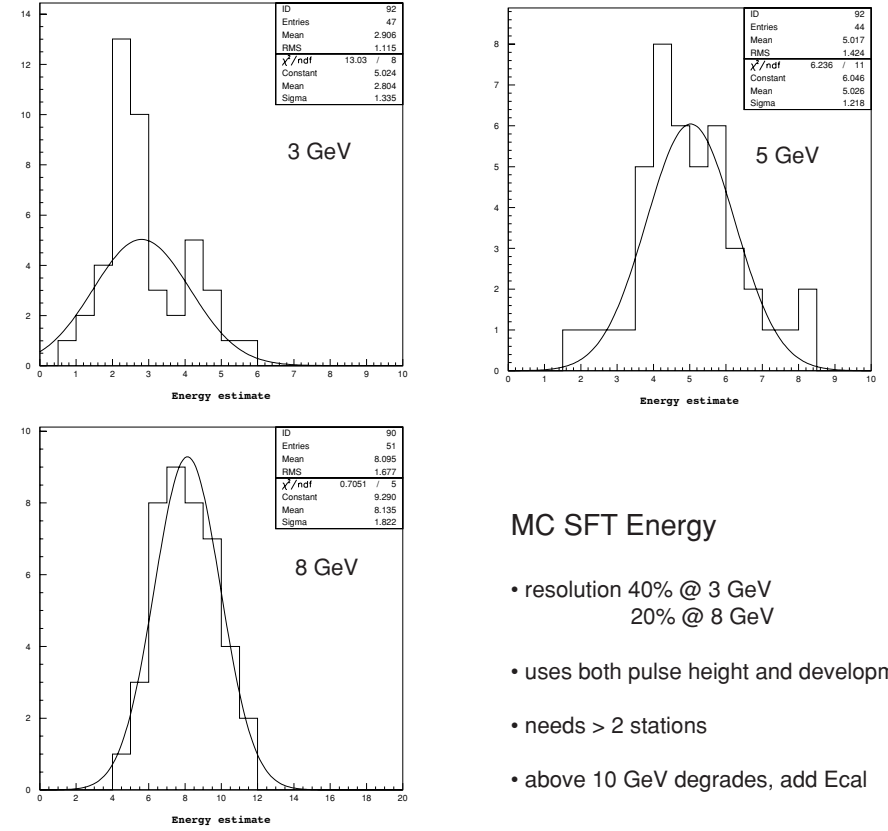
4

MC SFT Energy

8 GeV

ID	91
Entries	44
Mean	3.000
RMS	2.731

Sta 1

Nhits

ID	91
Entries	44
Mean	11.52
RMS	5.126

Sta 2

Nhits

ID	91
Entries	44
Mean	14.55
RMS	3.938

Sta 3

Nhits

ID	91
Entries	44
Mean	10.77
RMS	3.726

Sta 4

Nhits

Figure 3 The number of SFT hits for Monte Carlo 8 GeV electrons originating at *z*=0 for the four detector stations

ID	92
Entries	47
Mean	2.906
RMS	1.115
χ^2/ndf	13.03 / 8
Constant	5.024
Mean	2.804
Sigma	1.335

3 GeV

Energy estimate

ID	92
Entries	44
Mean	5.017
RMS	1.424
χ^2/ndf	6.236 / 11
Constant	6.046
Mean	5.026
Sigma	1.218

5 GeV

Energy estimate

ID	90
Entries	51
Mean	8.095
RMS	1.677
χ^2/ndf	0.7051 / 5
Constant	9.290
Mean	8.135
Sigma	1.822

8 GeV

Energy estimate

MC SFT Energy

• resolution 40% @ 3 GeV
    20% @ 8 GeV

• uses both pulse height and development

• needs > 2 stations

• above 10 GeV degrades, add Ecal

Figure 4 The estimated energy, E_{SFT}, for 3, 5, and 8 GeV electrons incident on Module 1 .

SFT Energy Calibration

Module 3 Verticies

Run	Event	E_{SFT}	E_{cal}	E_{cal}(est)	E_{SFT}+E_{cal}	ΔE	$\Delta E/E_{cal}$(est)
3202	8252	24.3	47.4	78.9±23.5	71.7	7.2	0.09
3223	41138	32.8	28.4	57.2±21.1	61.2	-4.0	-0.07
3232	18635	19.6	33.2	53.5±14.5	52.8	0.7	0.01
3278	17752	19.6	50.3	80.5±22.6	69.9	10.6	0.13
3359	8038	25.5	13.2	28.2±10.0	38.7	-10.5	-0.37

mean	-0.04
rms	0.18

Module 2 Verticies

Run	Event	E_{SFT}	E_{cal}	E_{cal}(est)	E_{SFT}+E_{cal}	ΔE	$\Delta E/E_{cal}$(est)
3059	1284	22.8	23.5	60.2	46.3	13.9	0.23
3062	10903	19.4	13.0	42.1	32.4	9.7	0.23
3113	27553	3.4	18.8	14.8	22.2	-7.4	-0.50
3130	28864	8.1	17.1	32.1	25.2	6.9	0.21
3202	6688	8.2	38.2	23.4	46.4	23.0	0.98
3240	9118	45.5	45.0	136.	90.5	45.5	0.33
3245	25897	24.7	45.7	130.	70.4	59.6	0.45
3298	17712	28.0	7.4	22.7	35.4	-12.7	-0.56
3299	9529	38.1	26.7	71.9	64.8	7.1	0.10
3315	17199	23.3	13.4	33.7	36.7	-3.0	-0.09

mean	0.14
rms	0.45

Table 1. Selected electron data from Period 3 or 4 with a vertex located in Module 3 (*top*) and Module 2 (*bottom*).
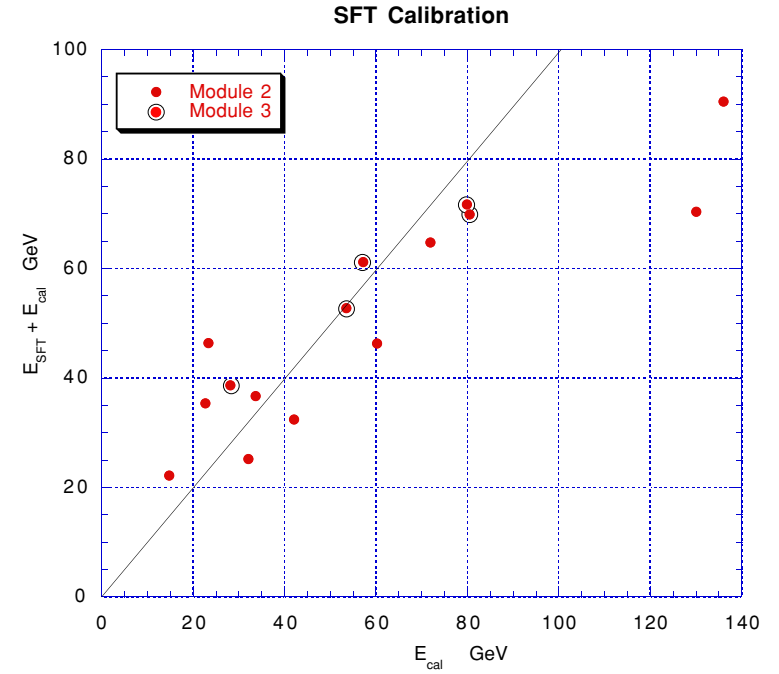

Figure 5. The data from Table 1, the estimated total energy as a function of the corrected calorimeter energy, E_{cal}. The data from Module 3 are circled.
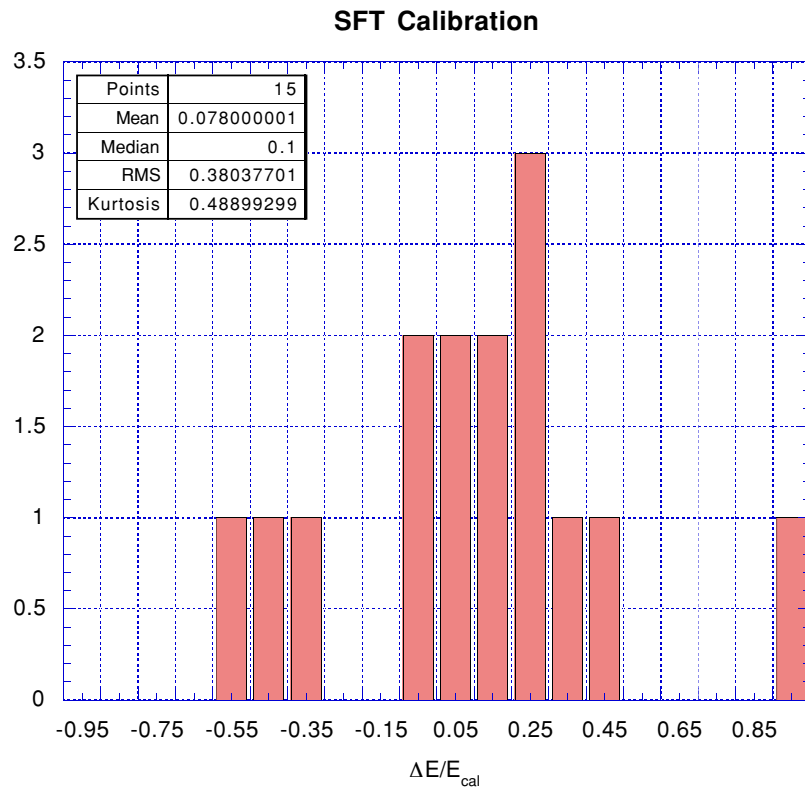
SFT Calibration

Points	15
Mean	0.078000001
Median	0.1
RMS	0.38037701
Kurtosis	0.48899299

$\Delta E/E_{cal}$

Figure 6. The distribution of the estimated difference in energy, $\Delta E = (E_{SFT}+E_{clus}) - E_{cal}$, divided by the estimated energy, E_{cal}.

Monte Carlo SFT and Calorimeter Distributions

Someof the parameterizations in this memo are derived from Monte Carlo data generated and analyzed in a previous memo, and here we reproduce the distributions for completeness. The electrons were generated at $z=0$ and with $p_{x,y}=0$. The energy deposited in the calorimeter from many sets of mono-energetic electrons was recorded as well as the number of hits and pulse heights in the SFTs.
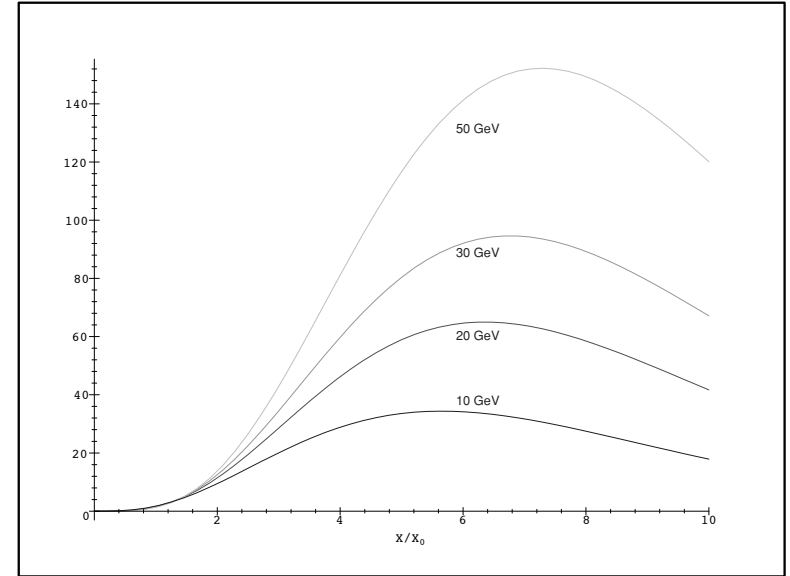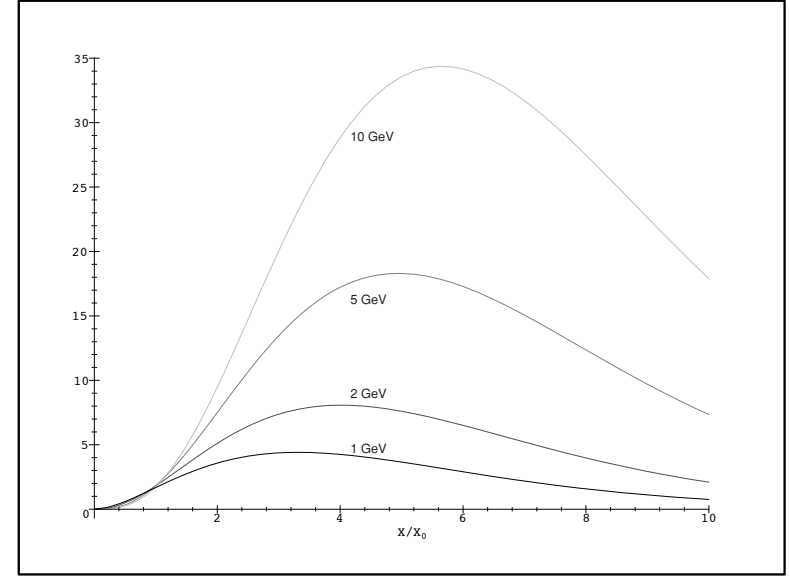
Figure A1. Functional approximations to the number of e^+/e^- in an electromagnetic shower versus radiation length. (Continued on following page).
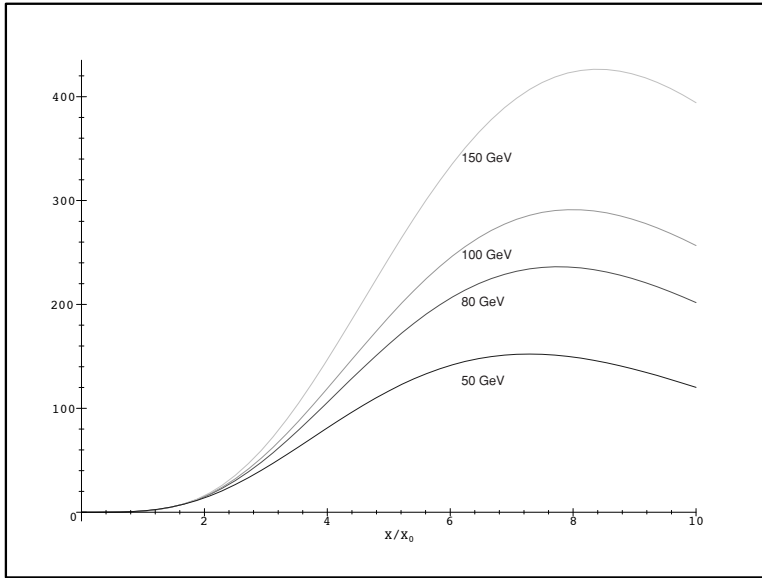
Figure A1 continued.
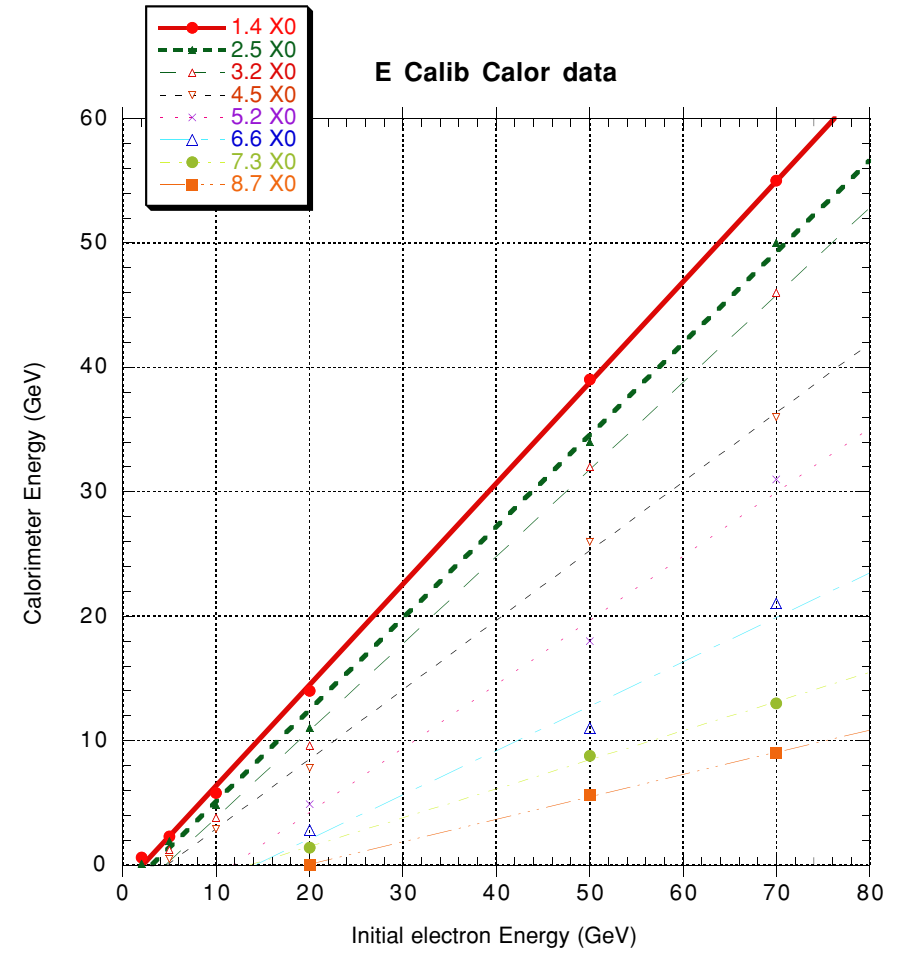
Figure A2. Energy visible in the calorimeter vs initial energy for various depths.
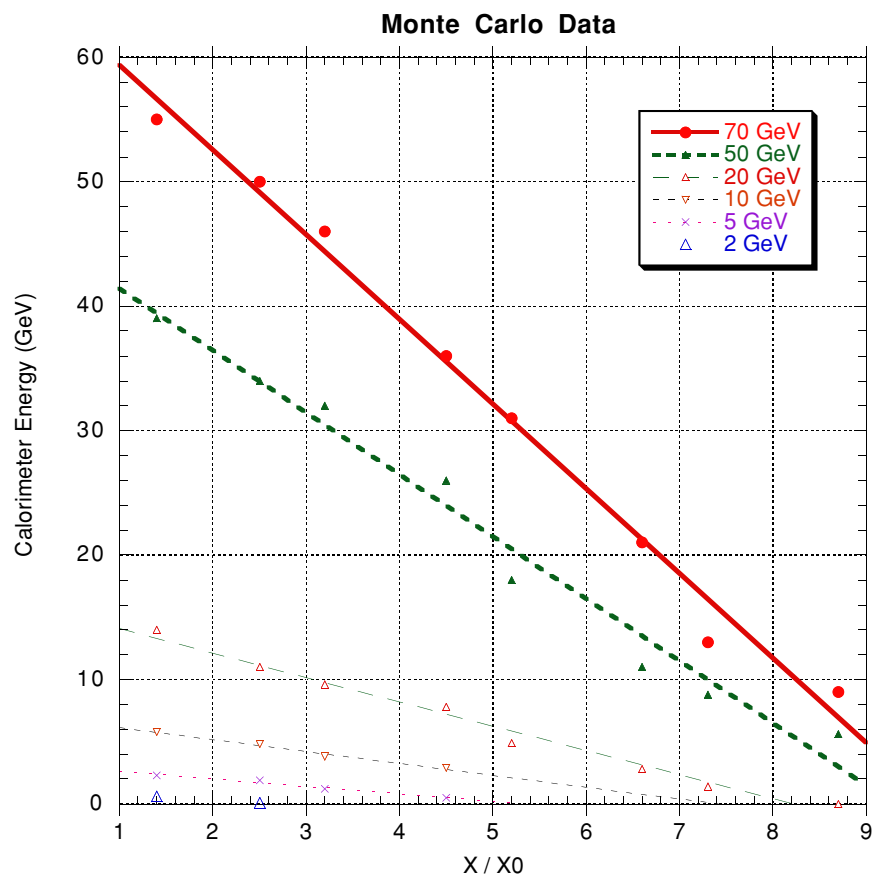
Figure A3. Calorimeter energy as a function of depth for various initial energies.
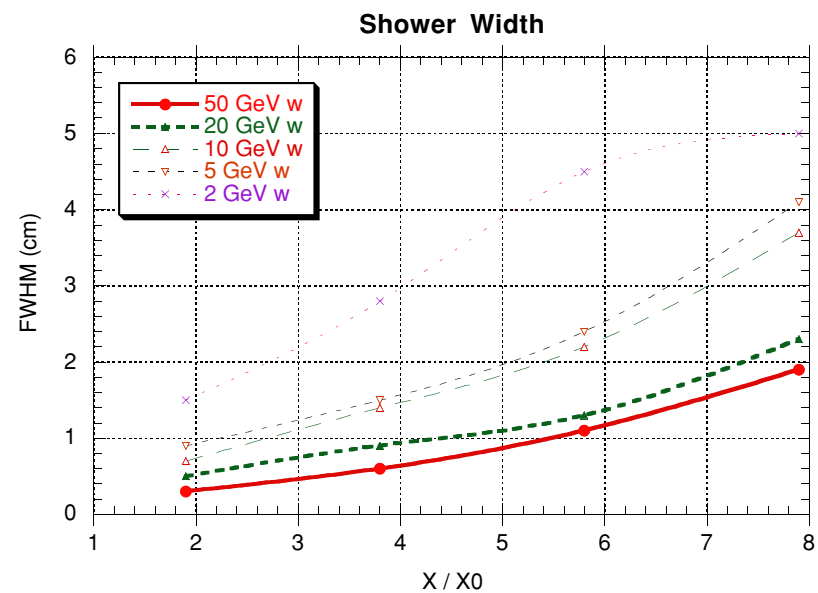

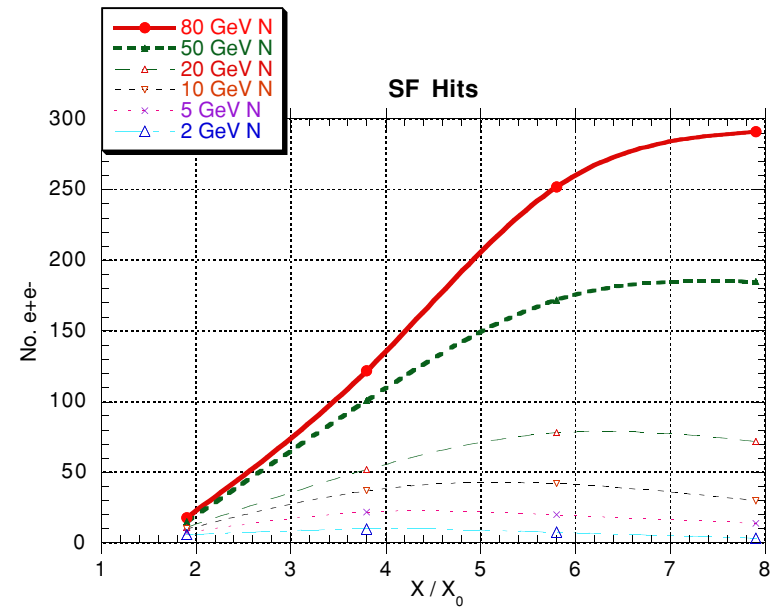**Figure A4.** Shower FWHM as a function of depth for various initial energies.

SF Hits (log scale plot, top left)

Legend:
- 80 GeV N
- 50 GeV N
- 20 GeV N
- 10 GeV N
- 5 GeV N
- 2 GeV N

Y-axis: No. e+e-
X-axis: X / X_0

MC Data

Legend:
- Est energy
- Add Ecal

Y-axis: Estimated energy (GeV)
X-axis: MC energy (GeV)

Figure A6. The estimated initial electron energy for MC data using the algorithm described in the text. Only SFT data is used except for one point which had significant energy in the calorimeter (greem triangle).

SF Hits (linear scale plot, bottom left)

Legend:
- 80 GeV N
- 50 GeV N
- 20 GeV N
- 10 GeV N
- 5 GeV N
- 2 GeV N

Y-axis: No. e+e-
X-axis: X / X_0

MC Data

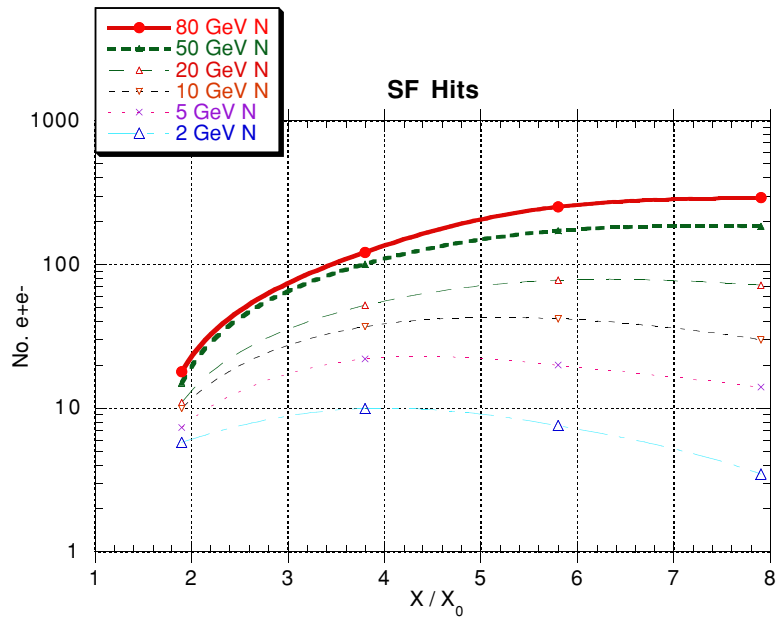| Points	1 9
Mean	0.066301738
RMS	0.17214501
Std Deviation	0.16321793

*not included

X-axis: ΔE (GeV)

Figure A5. Number of electron / positrons as a function of depth for varous initial energies. The same MC data is plotted on a log scale (*top*) and linear scale (*bottom*).
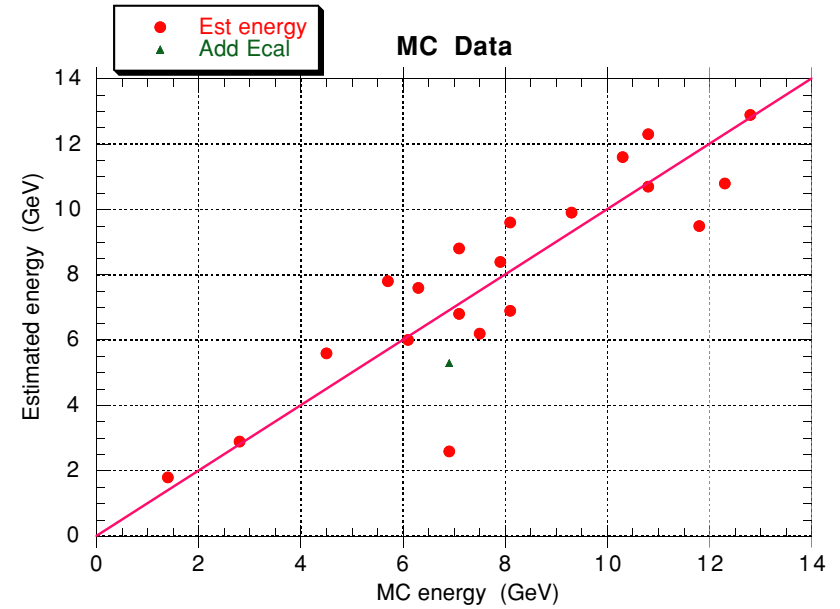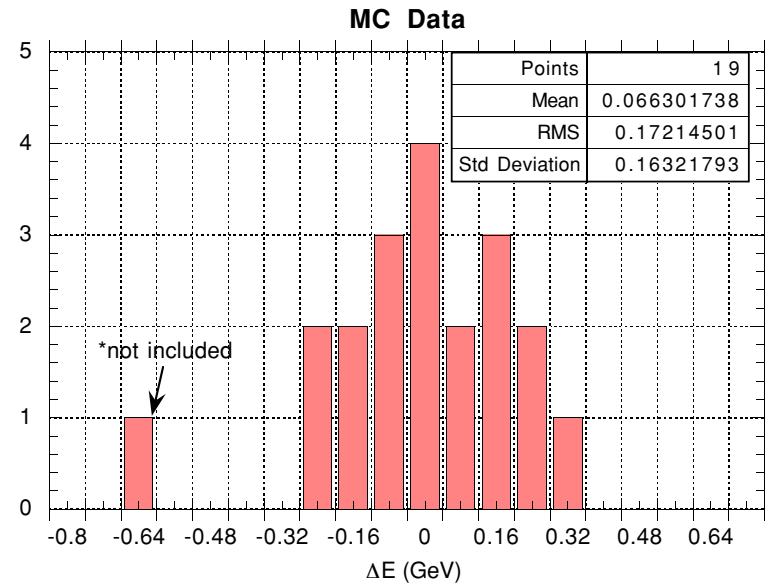
Figure A7. The difference, Eest - EMC, using the same algorithm as Figure A6. The leftmost datum corresponds to the low point in the above plot and was not included in the statistics.